Non-Faraday magnetically induced nonreciprocity in a fiber optic gyro.

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Abstract

Optical phase nonreciprocity may be caused by magnetically induced fiber mode deformation. This new effect sets up the fundamental limit of accuracy of the fiber optic gyro in the presence of magnetic field.

Introduction

Sagnac effect belongs to the wide set of nonreciprocal phenomena breaking symmetry of the counter-propagating waves. Other nonreciprocal effects may generate fiber optic gyro (FOG) erroneous signal. Faraday nonreciprocity (magnetically induced phase difference of counterpropagating waves) is of special importance due to the potentially large magnitude and due to the presence of magnetic field in most of FOG applications (Earth field ~1Gauss). Negligible magnetic sensitivity was expected for the FOG based on the birefringent polarization maintaining (PM) fiber because it works with linear polarized waves. However, since the fiber is twisted along its length the polarization modes are slightly elliptical and Faraday nonreciprocity is not fully suppressed. It's shown that Faraday magnetic response can be eliminated by equalizing intensities of the waves in both polarization modes. For the Fizoptika FOGs [1] this is feasible by adjustment of the azimuth of polarizer transmission axis. In the FOGs with suppressed Faraday response the non-Faraday magnetic response was detected. Its features do not correspond to the conventional effect, for instance:

- proportionality to the traverse projection of magnetic field;
- independence on polarizer azimuth.

To explain this effect we considered the fiber mode distribution in transverse magnetic field. It was shown that the field causes mode distortion equivalent to its lateral shift. The mode shift results in splitting of the counterpropagating wave trajectories and in their optical path lengths difference which is independent of the loop size and shape. Non-Faraday nonreciprocity may be potentially the major FOG error magnetically induced.

1. Faraday nonreciprocity

In the open loop FOG [1] all optical components are fabricated along single fiber length. Fiber birefringent axes are not aligned to simplify production technique. The transition coefficient of entire assembly **$T(\alpha_C,\alpha_P)$** normalized to the case $\alpha_C = \alpha_P = 0$ (aligned axes) is given by [2]

$$T(\alpha_C, \alpha_P) = ½ \{1 + \cos(2\alpha_C) \cos^2(2\alpha_P)\} \qquad (1)$$

Angle α_p is tunable by fiber turning near polarizing crystal. Magnetically induced phase nonreciprocity may be obtained [2] as:

$$\delta\varphi(\alpha_P, \alpha_C) = VHL \cos^2(\alpha_C) \cos(2\alpha_P) (t/b)\, 1/T(\alpha_C, \alpha_P) \qquad (2)$$

where $\delta\varphi\ (\alpha_P, \alpha_C)$ represents contribution to the phase nonreciprocity of the twisted fiber length L, V denotes Verdet constant, t- twist rate, b- birefringence, H - longitudinal projection of magnetic field over length L. To derive (2) it was assumed that polarization modes form two uncoupled sensing loops and that practically the condition **$b \gg t$** is fulfilled. Those loops acquire Faraday nonreciprocities of equal magnitudes but opposite signs. Integral Faraday response is minimized at $\alpha_P = 45^o$ when intensities of modes are equal.

To realize FOG with vanished magnetic sensitivity the FOG VG941-3AS was taken. To get signal of magnetic sensitivity its sensing coil was placed into solenoid (~100 Gauss) so, that solenoid horizontal axis was parallel to the sensing loop plane (Fig.1). Magnetic response was measured vs polarizer orientation (Fig.2 for **H_z**). Its magnitude was varying from sample to sample within the range 10 -30 deg/hr*Gauss because of random character of fiber twisting. The quasi-sine dependence corresponds to the theoretically predicted. After finding orientation with zero magnetic response, fiber azimuth α_P was fixed. Practically the residual magnetic sensitivity was about 1/50-1/30 from initial value due to temporal and temperature instability of α_P.

While checking magnetic response of adjusted FOGs in all three perpendicular directions we detected well reproducible (sample to sample) response ~3deg/hr*Gauss to the magnetic field directed along FOG's vertical sensitivity axis (i.e. orthogonal to the fiber axis). Moreover it was nearly independent on the polarizer orientation (Fig.2 for **H_y**). From collected data we found that the response is correlating mainly with the number of turns of the sensing loop with factor about 1 nano-radian/turn*Gauss. Observed magnetic response behavior can not be explained by Faraday magneto-optical effect.

2. Non-Faraday nonreciprocity

Counterpropagating waves in transverse magnetic field have identical conditions of propagation. Therefore their phase shifts difference may only occur if optical paths (trajectories) of the waves in the waveguide (fiber) are different. Such difference may be the result of magnetically induced separation of the optical paths of counter-propagating beams in the loop plane. The separation (arising from the opposite lateral mode shifts) does not give phase shifts difference in the straight fiber but in the curved fiber (as in the FOG's sensing loop) results in the phase nonreciprocity proportional to the beams separation.

It was theoretically shown that mode shift effects may be induced by fiber bent [7, 8] or fiber loop rotation [9]. Unlike those effects magnetically induced mode shift depends on the direction of propagation.

To analyze magnetic mode shift we find its field distribution in transverse ambient magnetic field $\mathbf{H = H_y}$ directed along FOG sensitivity axis. When $\mathbf{H = 0}$ the induction vector $\mathbf{D = n^2E}$ and wave equation has the form $\Delta\mathbf{E_0} + \mathbf{n^2k^2E_0 = 0}$ [10]. It is derived from Maxwell equations on condition $\nabla\mathbf{E_0 = 0}$ (negligible change of the refractive index over wavelength $\boldsymbol{\lambda}$ distance). We use the following coordinates: z- along fiber axis, x-axis lays in the loop plane and third y-axis. For the wave polarized in x-direction ($\mathbf{E_{ox} \neq 0, E_{oy} = 0}$) the electric field longitudinal z-component $\mathbf{E_{oz}}$ and its transverse component $\mathbf{E_{ox}}$ are bounded as follows (Fig.3)

$$\mathbf{E_{oz} = -\,(i/\beta)\partial E_{ox}/\partial x} \qquad (3)$$

For Gaussian approximation $\mathbf{E_{0x}(x,y) = exp\{-(x^2+y^2)/2w_0^2\}}$ ($\mathbf{w_0}$ denotes mode spot size) [7] the z-component of the field is

$$\mathbf{E_{oz}(x,y) = i(x/\beta w_0^2)E_{ox}(x,y)} \qquad (4)$$

In the presence of magnetic field induction vector is changed [6] to $\mathbf{D = n^2E + i(g\times E)}$ where $\mathbf{g = VH\lambda n/\pi}$ – gyration vector. The wave equation derived from Maxwell equations is then

$$\Delta\mathbf{E} + \mathbf{n^2k^2E} = \nabla(\nabla\mathbf{E}) - \mathbf{ik^2(g\times E)} \qquad (5)$$

equation for field x-component $\mathbf{E_x}$ within the first perturbation order

$$\Delta\mathbf{E_x} + \mathbf{n^2k^2E_x = -\,ig/n^2\cdot\partial^2E_{0z}/\partial x^2} \qquad (6)$$

for Gaussian mode approximation

$$\Delta\mathbf{E_x} + \mathbf{n^2k^2(1 + 3gx/k^3n^5w_0^4)\cdot E_x = 0} \qquad (7)$$

Magnetic field and guided mode z-component are the factors of distortion of the original wave equation. The perturbation is antisymmetrical along x-axis and hence brings nonsymmetrical distortion to the original mode field distribution. Let's note that z-component can not be neglected, as well as in the case of the fiber circular birefringence induced by its twisting [4]. For the counter-propagating waves the signs in parentheses in the equation (7) are opposite. This equation is similar to the wave equation for the curved fiber [7] which solution is known to be $\mathbf{E_x(x,y) = E_{0x}(x,y)(1+ax)}$. In our case coefficient $\mathbf{a = 3VH\lambda/2n^2w_0^2\pi k}$. Mode shift $\delta\mathbf{x}$ can be obtained by correspondent averaging of normalized field distribution $\mathbf{E_x(x,y)}$ over fiber cross section (Fig.4)

$$\delta\mathbf{x} = \iint \mathbf{dxdy\, xE_x(x,y)} \,/\, \iint \mathbf{dxdy|E_{0x}(x,y)|^2 = 2aw_0^2 = 3/2\pi^2\cdot VH\,(\lambda/n)^2} \qquad (8)$$

For quartz fiber δ**x** is about **10^{-5} Å/Gauss** (angstrom). Since Verdet constant **$V \sim 1/\lambda^2$ [**11**]** the shift magnitude is nearly not sensitive to waveguide and light parameters. For single fiber loop the phase difference $\delta\varphi$ between counterpropagating waves is **$4\pi\beta\delta x$**, and

$$\delta\varphi = 12HV\lambda/n \quad (\beta \approx kn) \qquad (9)$$

which is about 2 nanoradian/turn*Gauss. In a miniature FOG the number of turns exceeds 10^3, so the effect appears to be essential for some applications.

There is no magnetically induced mode shift for y-polarized wave E_{oy} as the vector product of the mode electric field and external magnetic field equals to zero. Birefringent axes of the fiber are not related to the x,y coordinates linked to the fiber loop plane. Actually due to fiber random twisting each of its birefringent axes coincides with x- axis along approximately half fiber length. Therefore actual magnetic response is about ½ of the calculated above (9) and it is nearly not sensitive to polarizer azimuth. The non-Faraday nonreciprocity is proportional to the number of turns and it is independent on size and shape of the fiber loop (Fig.5). This last statement is deduced from the fact that loop perimeter increment caused by lateral path shift is determined by the turn of the contour's normal which equals **2π** for any loop shape. Numerical estimation gives for FOG's error (any number of turns):

$$\delta\Omega \approx 4H/D^2 \qquad (10)$$

where the units are [$\delta\Omega$] = deg/hr, [H] = Gauss, [D] = cm. For instance, if D=2cm the bias error in Earth magnetic field is about 1 deg/hr.

3. Conclusions.

The new kind of magneto-optical nonreciprocity is established which unlike Faraday effect occurs in the transverse magnetic field. The cause of non-Faraday nonreciprocity is magnetically induced shift of the fiber mode. The shift results in separation of the optical paths of the counter-propagating waves. In a bent fiber this leads to a phase nonreciprocity which does not depend on sensing loop size and shape. Non-Faraday nonreciprocity sets fundamental limit of the FOG accuracy in the presence of magnetic field.

References

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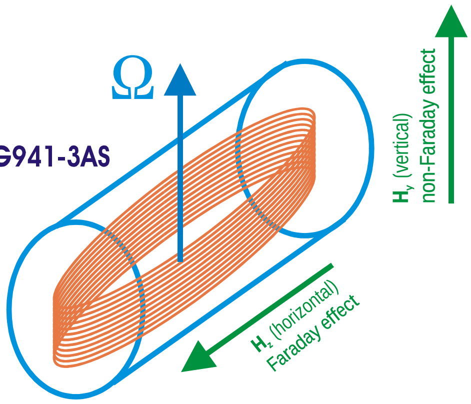


Fig.1 Setup configuration. When magnetic field is parallel to sensor's sensitivity axis (orthogonal to fiber axis) the non-Faraday nonreciprocity is observed.

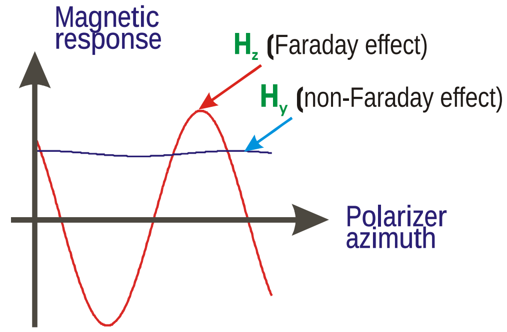


Fig.2 Dependence of sensor magnetic response on polarizer azimuth for Faraday nonreciprocity (red curve) and for non-Faraday nonreciprocity (blue curve)

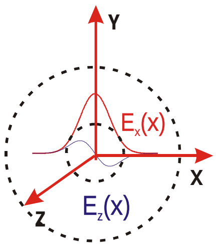


Fig.3 Coordinate system used. X-polarized horizontal field component $E_x(x)$ is in a loop plane. Field z-component $E_z(x)$ is anti-symmetrical function of x unlike symmetrical main field distribution.

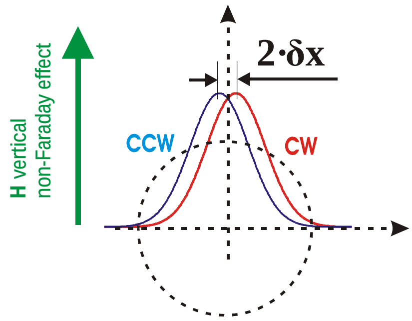
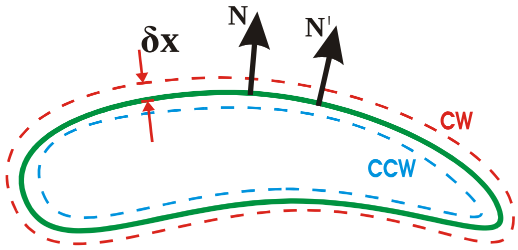


Fig.4 Distortion of x- polarized component generated by z-component in the presence of magnetic field appears as separation of field distributions of counter-propagating beams



$$\Delta l = 2\delta x \cdot \varphi(N,N')$$

$$\oint \varphi(N,N')dl = 2\pi$$

$$\Delta L = 4\pi \cdot \delta x$$

Fig.5 Separation of optical paths of counterpropagating beams results in a difference ΔL of loop optical lengths. This difference does not depend on loop shape and determined only by separation magnitude δx.